A Warm-start QAOA based approach using a swap-based mixer for the TSP

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1 Introduction

We examine quantum heuristics that utilize Mixer Hamiltonians, which facilitate the restriction of the search space to a specific subspace and support the implementation of warm-start strategies for solving the Traveling Salesman Problem (TSP). These approaches, based on Mixer Hamiltonians, can be integrated into the Quantum Approximate Optimization Algorithm (QAOA) [1], with the Mixer serving as a mapping function that transforms qubit strings into feasible solution sets. Initially, we introduce a swap-based mixer specifically designed for the TSP, ensuring that only qubit strings corresponding to valid TSP solutions are explored during the QAOA process. Subsequently, we present a warm-start technique that initializes QAOA with a solution derived from any classical heuristic, thereby facilitating faster convergence.

2 Problem definition and resolution

The Traveling Salesman Problem (TSP) is a vehicle routing problem where a set of customers must be visited by a vehicle, though a pickup or delivery is not necessarily required at each stop. The objective of this problem is to minimize the total distance traveled by the vehicle. The only available input data is the distance matrix $w_{j,k}$, which represents the distance between two customers j and k. Let T denote a tour in a TSP problem with n vertices. The tour T is defined by the sequence T_j which specifies the customer located at position j in the tour. The cost of the tour is then expressed as: $C(T) = \sum_{j=0}^{n} w_{T_j,T_{j+1}}$

The unconstraint formalization consists in defining an objective function that encompasses two extra penalization terms that are supposed to measure the violation of constraint using a weigh A and B that could be instance dependent [2].

$$C(x) = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1} w_{ij}\,.\sum_{p=1}^{n-2} x_{ip}.x_{j,p+1} + \sum_{i=0}^{n-1} w_{0,i}.x_{i,1} + \sum_{i=0}^{n-1} w_{i,0}.x_{i,n-1} + A.\sum_{p=1}^{n-1}\left(1-\sum_{i=1}^{n} x_{ip}\right)^2 + B.\sum_{i=1}^{n}\left(1-\sum_{p=1}^{n-1} x_{ip}\right)^2$$

The underlying principle involves defining an operator H_B that generates a superposition of all feasible solutions to the Traveling Salesman Problem (TSP), and only those solutions—i.e., qubit strings that satisfy all problem constraints. To construct this operator, we introduce the swap operator $Swap_{ij}$, which exchanges the states of two qubits. This operator is defined as follows:

$$Swap_{ij} = CX_{ij}.CX_{ji}.CX_{ij}$$

The implementation of the $Swap_{uv}$ of two nodes (which are encoded b several qubits) is based on the following theorem which gives an efficient way to define $Swap_{uv}$ as an unitary operator for the qubits used in the encoding of nodes.

Theorem 1. Definition of $e^{-i.t.Swap_{uv}}$

Since ${Swap_{uv}}^2 = Id$, we have $e^{-i.t.Swap_{uv}} = \cos t\,.Id - i.\sin t\,.Swap_{uv}$

Let $|\psi_0\rangle$ one auxiliary qubit and let us define define: $|\psi_0\rangle = \cos t\,.|0\rangle - i.\sin t\,.|1\rangle$

So

$$(CSwap_{uv})_0.|\psi_0,\psi_1,\dots,\psi_n\rangle = \cos t\,.|0,\psi_1,\dots,\psi_n\rangle - i.\sin t\,.|1\rangle \otimes Swap_{uv}.|\psi_1,\dots,\psi_n\rangle$$

By consequence the qubits 1 to n defined now:

$$(\cos t\,.Id - i.\sin t\,.Swap_{uv}).|\psi_1,\dots,\psi_{n-1},\psi_n\rangle = e^{-i.t.Swap_{uv}}.|\psi_1,\dots,\psi_{n-1},\psi_n\rangle$$

The numerical experiments meet the theoretical considerations enabling the exploration of qubit strings representing TSP solutions.

3 Concluding remarks

This paper introduces a swap-based mixer specifically tailored for the Traveling Salesman Problem (TSP), enabling the exclusive exploration of qubit strings that represent valid TSP solutions. We present numerical experiments that show the results are consistent with theoretical expectations. Such operator may open new possibilities for defining mixers, and when combined with a dedicated warm-start approach, it provides high-quality solutions

References

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