Decision-dependent information discovery for robust cardinality-constrained combinatorial optimization problems

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1 Introduction

Decision-Dependent Information Discovery (DDID), a variant of two-stage robust optimization, addresses problems where uncertainty can be reduced by querying the value of some uncertain parameters before making decisions for the nominal problem. The model has many applications in urban planning, project management, resource allocations, scheduling, among many others (see [8]). Recent works have focused on providing approximate solution algorithms based on K-adaptability [8], as well as exact algorithms considering either combinatorial Benders cuts [6] or even an exact monholitic reformulation [5], compact for specific uncertainty sets. Even more recently, an extension to distributionally robust optimization has been considered [4]. Despite these numerical approaches, very little is known about the complexity of specific problems under the DDID framework, with the exception of the recent paper [1] that considers the case of DDID applied to the selection problem and considering a discrete uncertainty set. This work intends to push further the understanding of the hardness of DDID, focusing on a single robust constraint. Specifically, we consider in this paper the setting of combinatorial optimization problems defined as follows.

Definition 1 *A combinatorial optimization problem* Π *on* $\{0,1\}^n$ *is a tuple* $\langle \mathcal{I}, f^{(I)}, \mathcal{Y}^{(I)} \rangle$*, where* $\mathcal{I}$ *is the instance set, and for each instance* $I \in \mathcal{I}$*,* $f^{(I)} : \{0,1\}^n \to \mathbb{R}$ *is the cost function and* $\mathcal{Y}^{(I)} \subseteq \{0,1\}^n$ *is the feasible solution set (or search space) over which* f *is to be optimized.*

Given a combinatorial optimization problem Π, without loss of generality, we base our discussion on an arbitrary instance of Π and omit the instance index for simplicity. Let $[n] = \{1, 2, \cdots, n\}$. We consider in this paper a generalization of the cardinality-constrained counterpart of problem Π, denoted by GenCard-Π, where the number of elements selected from a specified subset $S \subseteq [n]$ is limited. GenCard-Π can then be formulated as

$$\min_{y \in \mathcal{Y}} \left\{ c^T y \;\middle|\; \sum_{i \in S} y_i \leq b \right\}, \qquad \text{(GenCard-}\Pi\text{)}$$

where $c \in \mathbb{Z}^n_{\geq 0}$ is a cost vector and $b \in \mathbb{Z}$. Note that the coefficients in the cardinality constraint are 1 for elements in S and 0 for those in $[n] \setminus S$. Introducing uncertainty in these coefficients leads to the following robust counterpart

$$\min_{y \in \mathcal{Y}} \left\{ c^T y \;\middle|\; \xi^T y \leq b, \ \forall \xi \in \Xi \right\}, \qquad \text{(Robust-}\Pi\text{)}$$

where $\Xi = \left\{ \xi \in [0,1]^n \;\middle|\; \sum_{i \in [n]} \xi_i \leq \Gamma \right\}$ is a budgeted uncertainty set (e.g. [6]).

Applying DDID to Robust-Π, some of the components of ξ can be queried before choosing y; we thus obtain a problem with three levels of decisions: (i) the set of queried components $Q \in \mathcal{Q}$ is selected, (ii) the adversary reveals the exact values for the queried components with vector μ, (iii) the actual vector y is chosen, with the constraint that it must be feasible for all possible vectors ξ selected by the adversary. Defining $\mathcal{Q} = \{Q \subseteq [n] \mid |Q| = q\}$, the counterpart of Robust-Π with information discovery can be formulated as

$$\min_{Q\in\mathcal{Q}} \max_{\mu\in\Xi} \min_{y\in\mathcal{Y}} \left\{ c^T y \;\middle|\; \xi^T y \leq b,\ \forall \xi \in \Xi(Q,\mu) \right\}, \qquad \text{(DDID-Π)}$$

where

$$\Xi(Q,\mu) = \{\xi \in \Xi \mid \xi_i = \mu_i, \forall i \in Q\}$$

ensures that the revealed values μ for the queried components are consistent with the eventual adversarial choice ξ. In what follows, we denote the objective function of this problem as

$$\Phi(Q) = \max_{\mu\in\Xi} \min_{y\in\mathcal{Y}} \left\{ c^T y \;\middle|\; \xi^T y \leq b,\ \forall \xi \in \Xi(Q,\mu) \right\}.$$

1.1 Our contributions

It is easy to see that DDID-Π is at least as hard as GenCard-Π with $S = [n]$ since any instance of GenCard-Π with $S = [n]$ reduces to an instance of DDID-Π with $\Gamma = n$. Prior to this paper, it was, however, not clear how much harder DDID-Π is. First, the hardness of DDID-Π is established even when Π is defined as the selection problem, one of the easiest combinatorial problems, where, given n elements with costs and a positive integer p, one seeks a subset of p elements with minimum total cost. DDID-Selection is reducible from the partition problem; however, the full proof is lengthy and is therefore omitted in this extended abstract.

Next, we show that how to solve DDID-Π in polynomial time for any fixed q whenever GenCard-Π can be solved in polynomial time. The result follows from the observation that, given $Q \in \mathcal{Q}$, solving a polynomial number of GenCard-Π instances allows us to obtain a reduced feasible set $\tilde{\mathcal{Y}}(Q) \subseteq \mathcal{Y}$ of polynomial size. For problems Π with an exact-cardinality constraint, the construction of the corresponding set $\tilde{\mathcal{Y}}(Q)$ can be further improved by reducing the computational complexity and by providing an alternative algorithm. We then show how the optimality of a solution $y \in \tilde{\mathcal{Y}}(Q)$ can be verified by solving a polynomial number of linear programs of polynomial size. The proof of this result is omitted in this extended abstract, see Proposition 3 from Section 3. These results show that DDID-Selection, DDID-ShortestPath, DDID-SpanningTree, and DDID-Assignment, among others, can be solved in polynomial time when q is a constant. Overall, our main contributions are the following two theorems.

Theorem 1 *DDID-Selection is NP-hard.*

Theorem 2 *If there exists a polynomial-time algorithm for solving GenCard-Π, then DDID-Π can be solved in polynomial time when q is a constant.*

2 Reformulation of $\Phi(Q)$

Let us start by applying to our context a well-known reformulation from the robust combinatorial optimization literature, which states that any robust constraint under budget uncertainty can be reformulated as the disjunction of nominal constraints, see [2, 7]. To do so, we first introduce the notation

$$\varphi(Q,\mu) = \min_{y\in\mathcal{Y}} \left\{ c^T y \;\middle|\; \xi^T y \leq b,\ \forall \xi \in \Xi(Q,\mu) \right\}, \qquad (1)$$

so $\Phi(Q) = \max_{\mu \in \Xi} \varphi(Q, \mu)$. Let us consider the reformulation

$$\varphi'(Q,\mu) = \min_{y \in \mathcal{Y}} \left\{ c^T y \;\middle|\; \sum_{i \in Q} \mu_i y_i + \min\left\{ \sum_{i \in \bar{Q}} y_i,\ \Gamma - \sum_{i \in Q} \mu_i \right\} \leq b \right\}. \tag{2}$$

Lemma 1 *For each $Q \in \mathcal{Q}$ and $\mu \in \Xi$, $\varphi'(Q,\mu) = \varphi(Q,\mu)$.*

Proof : Observe that for all $y \in \mathcal{Y}$ and $\xi \in \Xi(Q,\mu)$, $\xi^T y = \sum_{i \in Q} \mu_i y_i + \sum_{i \in \bar{Q}} \xi_i y_i \leq \sum_{i \in Q} \mu_i y_i + \min\left\{ \sum_{i \in \bar{Q}} y_i,\ \Gamma - \sum_{i \in Q} \mu_i \right\}$, so $\varphi'(Q,\mu) \geq \varphi(Q,\mu)$.

To prove the reverse inequality, let y^* be an optimal solution of the rhs of (1) and $\xi^* \in \arg\max_{\xi \in \Xi(Q,\mu)} \xi^T y^*$. We prove next that $\sum_{i \in Q} \mu_i y_i^* + \min\left\{ \sum_{i \in \bar{Q}} y_i^*,\ \Gamma - \sum_{i \in Q} \mu_i \right\} \leq b$ by considering two cases. If $\sum_{i \in \bar{Q}} y_i^* \leq \Gamma - \sum_{i \in Q} \mu_i$, then $\xi_i^* = 1$ for each $i \in \bar{Q}$ so $\sum_{i \in Q} \mu_i y_i^* + \sum_{i \in \bar{Q}} y_i^* = \xi^{*T} y^* \leq b$. Otherwise, there is a subset $S \subseteq \bar{Q}$ of size $\lceil \Gamma - \sum_{i \in Q} \mu_i \rceil$ such that $y_i^* = 1$ for each $i \in S$, and $\xi_i^* = 1$ for each $i \in S \setminus \{s\}$ (for some $s \in S$) while $\xi_s^* = \Gamma - \sum_{i \in Q} \mu_i - \lfloor \Gamma - \sum_{i \in Q} \mu_i \rfloor$. In this case $\sum_{i \in Q} \mu_i y_i^* + \Gamma - \sum_{i \in Q} \mu_i = \xi^{*T} y^* \leq b$. Therefore, y^* is feasible for the rhs of (2) so $\varphi'(Q,\mu) \leq \varphi(Q,\mu)$, proving the result. $\square$

3 Polynomial-time solution for constant q

In this section, we present an algorithm that solves DDID-Π in polynomial time for certain nominal problems Π, provided that q, the size of the queried set, is a constant.

3.1 Proof of Theorem 2

With constant q, DDID-Π can be solved by enumerating all elements of $\mathcal{Q}$, whose size is polynomial in n (since $|\mathcal{Q}| = \binom{n}{q} \in O(n^q)$). Consequently, if the function $\Phi(Q)$ is computable in polynomial time, DDID-Π can be solved in polynomial time. We begin by eliminating dominated solutions $y \in \mathcal{Y}$ with respect to the minimization problem defining $\varphi(Q,\mu)$.

Observation 1 *For any given $\mu \in \Xi$, let $y, \tilde{y} \in \mathcal{Y}$ such that $y_i \leq \tilde{y}_i, \forall i \in Q$ and $\sum_{i \in \bar{Q}} y_i \leq \sum_{i \in \bar{Q}} \tilde{y}_i$. If $c^T y \leq c^T \tilde{y}$, then $\tilde{y}$ is dominated by y with respect to the minimization problem defining $\varphi(Q,\mu)$.*

Let $y_Q \in \{0,1\}^q$ denote the components corresponding to Q. To eliminate dominated solutions described in Observation 1, we solve the following optimization problem for each partial solution $\tilde{y}_Q \in \{0,1\}^q$ and each cardinality $k \in [n]$, which leads to solving instances of GenCard-Π:

$$\min_{y \in \mathcal{Y}} \left\{ \sum_{i \in \bar{Q}} c_i y_i + M \sum_{i \in Q : \tilde{y}_i = 0} y_i \;\middle|\; \sum_{i \in \bar{Q}} y_i \leq k \right\}, \tag{3}$$

with $M = \sum_{i \in [n]} c_i + 1$. Let $y^*(\tilde{y}_Q, k)$ denote an optimal solution of (3) and $z^*(\tilde{y}_Q, k)$ its optimal cost, define the reduced feasibility set as

$$\tilde{\mathcal{Y}}(Q) = \bigcup_{\substack{\tilde{y}_Q \in \{0,1\}^q,\ k \in [n]: \\ z^*(\tilde{y}_Q,k) < M}} \{ y^*(\tilde{y}_Q, k) \},$$

and let us introduce

$$\tilde{\varphi}(Q,\mu) = \min_{y \in \tilde{\mathcal{Y}}(Q)} \left\{ c^T y \;\middle|\; \sum_{i \in Q} \mu_i y_i + \min\left\{ \sum_{i \in \bar{Q}} y_i,\ \Gamma - \sum_{i \in Q} \mu_i \right\} \leq b \right\}.$$

Lemma 2 *For each $Q \in \mathcal{Q}$ and $\mu \in \Xi$, $\tilde{\varphi}(Q,\mu) = \varphi(Q,\mu)$.*

Proof : The inclusion $\tilde{\mathcal{Y}}(Q) \subseteq \mathcal{Y}$ immediately yields $\tilde{\varphi}(Q,\mu) \geq \varphi(Q,\mu)$. To prove the reverse inequality we denote by y' an optimal solution to the minimization problem defining $\varphi(Q,\mu)$ over $\mathcal{Y}$, and define $\tilde{y} = y^*(y'_Q, \sum_{i\in\bar{Q}} y'_i) \in \tilde{\mathcal{Y}}(Q)$. By construction, we have $\{i \in Q \mid \tilde{y}_i = 1\} \subseteq \{i \in Q \mid y'_i = 1\}$ and $\sum_{i\in\bar{Q}} \tilde{y}_i \leq \sum_{i\in\bar{Q}} y'_i$. Since y' is a feasible solution for the minimization problem defining $\varphi(Q,\mu)$, it follows that $\sum_{i\in Q} \mu_i \tilde{y}_i + \min\left\{\sum_{i\in\bar{Q}} \tilde{y}_i,\ \Gamma - \sum_{i\in Q}\mu_i\right\} \leq b$, so $\tilde{y} \in \tilde{\mathcal{Y}}(Q)$ is also feasible for the problem. Therefore, $\tilde{\varphi}(Q,\mu) \leq c^T\tilde{y} \leq \sum_{i\in\bar{Q}} c_i\tilde{y}_i + \sum_{i\in Q: y'_i=1} c_i \leq \sum_{i\in\bar{Q}} c_i y'_i + \sum_{i\in Q: y'_i=1} c_i = c^T y' = \varphi(Q,\mu)$. □

Remark 1 $|\tilde{\mathcal{Y}}(Q)| \in O(2^q n)$.

We now compute the value of $\Phi(Q) = \max_{\mu\in\Xi} \varphi(Q,\mu)$. First, observe that, if there exists some $\mu \in \Xi$ such that $\sum_{i\in Q} \mu_i y_i + \min\left\{\sum_{i\in\bar{Q}} y_i,\ \Gamma - \sum_{i\in Q}\mu_i\right\} > b$ for all $y \in \tilde{\mathcal{Y}}(Q)$, then $\Phi(Q) = +\infty$.

Proposition 1 *The boundedness of $\Phi(Q)$ can be verified by solving $O(n)$ linear programs, each with $O(q)$ variables and $O(2^q n)$ constraints.*

Proof : Omitted, due to space restrictions. □

When $\Phi(Q)$ is bounded, we next show that, among all $y \in \tilde{\mathcal{Y}}(Q)$ whose cost achieves the value of $\varphi(Q,\mu)$ for some $\mu \in \Xi$, the one with the highest cost determines $\Phi(Q)$. Let us formalize this idea by introducing some notations. For any given $\mu \in \Xi$, we define $\tilde{\mathcal{Y}}(Q,\mu) = \left\{y \in \tilde{\mathcal{Y}}(Q) \;\middle|\; \sum_{i\in Q}\mu_i y_i + \min\left\{\sum_{i\in\bar{Q}} y_i,\ \Gamma - \sum_{i\in Q}\mu_i\right\} \leq b\right\}$ as the set of all feasible solutions y under given μ. Furthermore, for any given $\hat{y} \in \tilde{\mathcal{Y}}(Q)$, we define $\hat{\Xi}(Q,\hat{y}) = \left\{\mu \in \Xi \;\middle|\; \hat{y} \in \arg\min_{y\in\tilde{\mathcal{Y}}(Q,\mu)} c^T y\right\}$ as the set of scenarios $\mu \in \Xi$ under which the solution $\hat{y}$ is optimal to the minimization problem defining $\varphi(Q,\mu)$. Last, we define the set $\hat{\mathcal{Y}}(Q) = \left\{y \in \tilde{\mathcal{Y}}(Q) \;\middle|\; \hat{\Xi}(Q,y) \neq \emptyset\right\}$.

Proposition 2 *If $\Phi(Q)$ is bounded, then $\Phi(Q) = \max_{y\in\hat{\mathcal{Y}}(Q)} c^T y$.*

Proof : We prove that

$$\max_{\mu\in\Xi} \min_{y\in\tilde{\mathcal{Y}}(Q,\mu)} c^T y = \max_{y\in\hat{\mathcal{Y}}(Q)} c^T y. \tag{4}$$

First, let μ^* be an optimizer of the lhs of (4) and let $y^* \in \arg\min_{y\in\tilde{\mathcal{Y}}(Q,\mu^*)} c^T y$. Thus, $\mu^* \in \hat{\Xi}(Q,y^*)$ so that $y^* \in \hat{\mathcal{Y}}(Q)$, and $\max_{\mu\in\Xi} \min_{y\in\tilde{\mathcal{Y}}(Q,\mu)} c^T y = c^T y^* \leq \max_{y\in\hat{\mathcal{Y}}(Q)} c^T y$.

Conversely, let y^{**} be an optimizer of the rhs of (4). This implies that $\hat{\Xi}(Q,y^{**}) \neq \emptyset$ so there exists

$$\mu^{**} \in \hat{\Xi}(Q,y^{**}) \subseteq \Xi \tag{5}$$

and, furthermore,

$$y^{**} \in \arg\min_{y\in\tilde{\mathcal{Y}}(Q,\mu^{**})} c^T y. \tag{6}$$

Therefore, $\max_{y\in\hat{\mathcal{Y}}(Q)} c^T y = c^T y^{**} = \min_{y\in\tilde{\mathcal{Y}}(Q,\mu^{**})} c^T y \leq \max_{\mu\in\Xi} \min_{y\in\tilde{\mathcal{Y}}(Q,\mu)} c^T y$, where the second equality follows from (6) and the inequality follows from (5).

□

Now, let us discuss how to determine whether a given solution $y \in \tilde{\mathcal{Y}}(Q)$ belongs to $\hat{\mathcal{Y}}(Q)$. It suffices to verify whether the (non-necessarily closed) set $\hat{\Xi}(Q, y)$ is non-empty. Let us first reformulate $\hat{\Xi}(Q, y)$ as:

$$\hat{\Xi}(Q,y) = \left\{\mu \in \Xi \;\middle|\; y \in \tilde{\mathcal{Y}}(Q,\mu),\; y' \notin \tilde{\mathcal{Y}}(Q,\mu), \forall y' \in \tilde{\mathcal{Y}}(Q) : c^T y' < c^T y\right\}$$

$$= \left\{\mu \in \Xi \;\middle|\; \sum_{i \in Q} \mu_i y_i + \min\left\{\sum_{i \in \bar{Q}} y_i, \Gamma - \sum_{i \in Q} \mu_i\right\} \le b, \right. \tag{7}$$

$$\left. \sum_{i \in Q} \mu_i y_i' + \min\left\{\sum_{i \in \bar{Q}} y_i', \Gamma - \sum_{i \in Q} \mu_i\right\} > b,\; \forall y' \in \tilde{\mathcal{Y}}(Q) : c^T y' < c^T y \right\} \tag{8}$$

Proposition 3 *The nonemptiness of the set $\hat{\Xi}(Q, y)$ can be verified by solving $O(2^q n^2)$ linear programs, each with $O(q)$ variables and $O(2^q n)$ constraints.*

Proof : Omitted, due to space restrictions. □

Based on Proposition 2, we propose an algorithm for computing the function $\Phi(Q)$, which is described as follows:

Algorithm 1: Compute $\Phi(Q)$

1 Find the set $\tilde{\mathcal{Y}}(Q)$ by solving $O(2^q n)$ instances of GenCard-Π .
2 Verify the boundedness of $\Phi(Q)$ by solving $O(n)$ linear programs of polynomial size.
3 Sort the solutions in $\tilde{\mathcal{Y}}(Q)$ in non-increasing order of their costs.
4 For each sorted solution in $\tilde{\mathcal{Y}}(Q)$, verify whether it belongs to $\hat{\mathcal{Y}}(Q)$ by solving $O(n)$ linear programs of polynomial size. The cost of the first solution encountered that belongs to $\hat{\mathcal{Y}}(Q)$ is then the value of $\Phi(Q)$.

Remark 2 *Algorithm 1 runs in $O(2^q n \cdot f(I) + 2^q n\Gamma \cdot g(n)) = O(2^q n \cdot f(I) + 2^q n^2 \cdot g(n))$ time, where $f(I)$ denotes the complexity of solving instance I of GenCard-Π, and $g(n)$ denotes the complexity of solving a linear problem with $O(q)$ variables and $O(2^q n)$ constraints.*

When q is a constant, DDID-Π can be solved by enumerating all elements in $\mathcal{Q}$, whose size is polynomial in n. Each $Q \in \mathcal{Q}$ can be evaluated using Algorithm 1, which runs in polynomial time whenever GenCard-Π is polynomially solvable. Theorem 2 is thus proved.

3.2 Exact-cardinality constraint

We can improve our results for the case of a problem Π for which an exact cardinality-constrained, whether implicit or explicit, must be satisfied. In this case, we show that we can either reduce the number of GenCard-Π instances that need to be solved or, alternatively, replace the generalized cardinality constraints by imposing $y_i = 1$ for all elements selected in the partial solution $\tilde{y}_Q$.

Proposition 4 *If problem Π has an (explicit or implicit) exact-cardinality constraint ($\sum_{i \in [n]} y_i = d$), then $\tilde{\mathcal{Y}}(Q)$ can be obtained by solving $O(2^q)$ instances of GenCard-Π, each formulated as in (3) for $\tilde{y}_Q \in \{0,1\}^q$ with $k = d - \sum_{i \in Q} \tilde{y}_i$.*

Proof : For any fixed $\tilde{y}_Q$, if $k < d - \sum_{i \in Q} \tilde{y}_i$, there is no solution $y \in \mathcal{Y}$; whereas if $k > d - \sum_{i \in Q} \tilde{y}_i$, two cases arise: either the obtained solution coincides with that for $k = d - \sum_{i \in Q} \tilde{y}_i$, or a cheaper solution y' is found, which can also be obtained when consider another instance with the pair $(y'_Q, d - \sum_{i \in Q} y'_i)$.

□

Observe that any optimal solution $y^*(\tilde{y}_Q, k) \in \tilde{\mathcal{Y}}(Q)$, obtained by solving an instance defined in Proposition 4, satisfies $y^*_i = \tilde{y}_i$ for all $i \in Q$. Therefore, one can equivalently solve the following problem instead:

$$\min_{y \in \mathcal{Y}} \left\{ \sum_{i \in \bar{Q}} c_i y_i + M \sum_{i \in Q : \tilde{y}_i = 0} y_i \;\middle|\; y_i = 1, \forall i \in Q : \tilde{y}_i = 1 \right\}. \tag{9}$$

Remark 3 *If problem* Π *has an (explicit or implicit) exact-cardinality constraint (*$\sum_{i \in [n]} y_i = d$*), then* $|\tilde{\mathcal{Y}}(Q)|$ *reduces to* $O(2^q)$*, and Algorithm 1 consequently runs in* $O(F(I) + 2^q \Gamma \cdot g(n))$ *time, where* $F(I)$ *denotes the complexity of computing the set* $\tilde{\mathcal{Y}}(Q)$ *for instance* I*.*

3.3 Example

As an illustration of Theorem 2, we show that DDID-ShortestPath can be solved in polynomial time when q is constant.

Example 1 *(Shortest Path) Given a directed edge-weighted graph* $G = (V, E)$ *and two vertices* $s, t \in V$*, the shortest path problem asks for a minimum-cost path from* s *to* t*. The GenCard-ShortestPath problem is obviously a special case of capacity-constrained shortest path problem, in which each edge* $e \in E$ *is associated with a capacity consumption* t_e*, and a capacity limit* T *is given. A dynamic programming approach for solving such problems has been introduced, see [3], which runs in* $O(|E|T)$ *time. In our case, we have* $t_e = 0$ *for all* $e \in Q$*,* $t_e = 1$ *for all* $e \in \bar{Q}$*, and* $T \leq |E|$*, GenCard-ShortestPath can thus be solved in* $O(|E|^2)$*. Hence, when* q *is a constant, DDID-ShortestPath can be solved in polynomial time.*

References

[1] Xiaoyu Chen, Marc Goerigk, and Michael Poss. The robust selection problem with information discovery. *arXiv preprint arXiv:2501.02510*, 2025.

[2] Kai-Simon Goetzmann, Sebastian Stiller, and Claudio Telha. Optimization over integers with robustness in cost and few constraints. In *International Workshop on Approximation and Online Algorithms*, pages 89–101. Springer, 2012.

[3] Refael Hassin. Approximation schemes for the restricted shortest path problem. *Mathematics of Operations Research*, 17(1):36–42, 1992.

[4] Qing Jin, Angelos Georghiou, Phebe Vayanos, and Grani A Hanasusanto. Distributionally robust optimization with decision-dependent information discovery. *arXiv preprint arXiv:2404.05900*, 2024.

[5] Jérémy Omer, Michael Poss, and Maxime Rougier. Combinatorial robust optimization with decision-dependent information discovery and polyhedral uncertainty. *Open Journal of Mathematical Optimization*, Volume 5(5):1–25, 2024.

[6] Rosario Paradiso, Angelos Georghiou, Said Dabia, and Denise Tönissen. Exact and approximate schemes for robust optimization problems with decision dependent information discovery. *INFORMS Journal on Computing*, 2024. In press.

[7] Artur Alves Pessoa, Michael Poss, Ruslan Sadykov, and François Vanderbeck. Branch-cut-and-price for the robust capacitated vehicle routing problem with knapsack uncertainty. *Operations Research*, 69(3):739–754, 2021.

[8] Phebe Vayanos, Angelos Georghiou, and Han Yu. Robust optimization with decision-dependent information discovery. *Management Science*, 2024. In press.