The robust time-dependent vehicle routing problem with time windows and budget uncertainty

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1 Introduction

The vehicle routing problem (VRP) with time windows (VRPTW) is an important variant of the VRP class, with many real-world applications in logistics and supply chain management. In the VRPTW, the goal is to design least-cost routes starting and ending at a single depot, ensuring that all customers are visited exactly once while respecting the vehicle's capacity and time windows imposed by the customers.

To provide more precise route schedules, the VRPTW models may consider time dependency in the travel time. The time-dependent VRPTW (TDVRPTW) considers the travel time as a function of the instant departure time (Gendreau et al., 2015). As a result, these functions are sensible to congestion variance during the day.

In practical applications, the real traffic data used as input to the optimizers are often obtained by location providers that predict the travel times for specified timeframes. The predictors estimate them based on historical data, where the traffic flow patterns are captured, increasing/decreasing the travel time in peak/off-peak hours. Furthermore, these providers enhance their forecasts by monitoring the live traffic and updating the travel time throughout the day. This may add unpredictable delays from uncertainties such as accidents, weather conditions, and road maintenance.

This work considers the problem where the planning of the routes must be designed one day in advance or within hours before execution in a static fashion and with fixed paths. We propose a novel framework to build routing schedules resistant to uncertainties based on *here-and-now* decisions taken before starting their operation. Hence, we introduce the robust time-dependent VRPTW (RTDVRPTW), a variant of the VRPTW that tackles the uncertainty of the time-dependent travel time using robust optimization. Our model extends the classical budget uncertainty set from Bertsimas and Sim (2004) to this setting.

Contributions Our study provides both numerical evidence of the relevance of the RTDVRPTW model and theoretical insights into its solution. Using real data, we show that RTDVRPTW yields better reliability–cost trade-offs than robust models ignoring time dependence. Addressing key questions for heuristic and exact methods, such as feasibility checking via dynamic programming, led us to develop three exact approaches: a compact formulation, a scenario-generation method, and a branch-and-cut scheme with lazy separation. We also implement an iterated local search (ILS) algorithm to produce high-quality primal bounds. Due to space limits, we detail only the compact formulation, which, although less efficient, is the most original of the three and shows that the decision version of the problem lies in $\mathcal{NP}$.

2 Problem definition

The TDVRPTW is defined on a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V} = \mathcal{C} \cup \{o, d\}$, $\mathcal{C}$ is the set of customers, o and d are the origin and destination depots. Each vertex $i \in \mathcal{V}$ has a demand q_i, and a time window $[e_i, l_i]$, with $q_o = q_d = 0$, and $[e_o, l_o] = [e_d, l_d] = [0, H]$, for a time horizon H. Set $\mathcal{A} = \{(i,j) \in \mathcal{C}^2 : i \neq j\} \cup \{(o,j) : j \in \mathcal{C}\} \cup \{(i,d) : i \in \mathcal{C}\}$ contains arcs between pairs of distinct customers, the origin depot and customers, and customers and the destination depot. Each arc has a travel cost c_{ij} and a time-dependent travel time function $t_{ij}(s)$, where $s \in \mathbb{R}_+$ is the departure time. An unlimited homogeneous fleet with capacity Q is assumed.

Our RTDVRPTW model decomposes the travel time of each arc into two functions, $\bar{t}_{ij}(s)$ and $\hat{t}_{ij}(s)$, denoting the nominal travel time and the deviation for the arc $(i,j) \in \mathcal{A}$ at $s \in \mathbb{R}_+$, respectively. We hereafter assume that the functions $\bar{t}_{ij}(s)$ and $\hat{t}_{ij}(s)$ are known and piecewise linear, with P breakpoints in the sets $\bar{B}_{ij} = \{(\bar{\mathfrak{s}}^1_{ij}, \bar{\mathfrak{t}}^1_{ij}), \ldots, (\bar{\mathfrak{s}}^P_{ij}, \bar{\mathfrak{t}}^P_{ij})\}$ and $\hat{B}_{ij} = \{(\hat{\mathfrak{s}}^1_{ij}, \hat{\mathfrak{t}}^1_{ij}), \ldots, (\hat{\mathfrak{s}}^P_{ij}, \hat{\mathfrak{t}}^P_{ij})\}$, respectively. Note that we assume, without loss of generality and to simplify the notation, that all functions have exactly P breakpoints. Given a nonnegative integer Γ, each scenario in our robust model then assigns each travel time function to $t_{ij}(s, \delta) = \bar{t}_{ij}(s) + \delta_{ij}\hat{t}_{ij}(s)$ for some binary vector δ in the set

$$\Delta_\Gamma = \left\{ \delta \in \{0,1\}^{|\mathcal{A}|} : \sum_{(i,j)\in\mathcal{A}} \delta_{ij} \leq \Gamma \right\}.$$

Most of our results assume that the *first-in-first-out* (FIFO) assumption holds, which is formally stated below.

Assumption 1. *$\partial_+\bar{t}_{ij}(s) > -1$ and $\partial_+\hat{t}_{ij}(s) > -1$ for each $(i,j) \in \mathcal{A}$ and $s \in \mathbb{R}_+$.*

The RTDVRPTW aims to determine a set of least-cost paths starting at o and ending at d, ensuring that each customer is visited within its time window, while following a time-dependent travel time function across all scenarios in Δ_Γ, and satisfying vehicle capacity constraints.

We now provide a mathematical formulation for the problem when Assumption 1 holds, in which case waiting at some node i more than necessary to reach e_i is never beneficial to satisfy the time windows. Let x_{ij} be a binary variable that is equal to 1 iff a vehicle traverses arc $(i,j) \in \mathcal{A}$, w_i be a nonnegative real variable indicating the load at vertex $i \in \mathcal{V}$, and $y_i(\delta)$ be the arrival time at vertex i for a given uncertain vector $\delta \in \Delta_\Gamma$. In addition, let T_{ij} be sufficiently large coefficients. We formulate the RTDVRPTW as follows:

$$\min \sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ij} \tag{1}$$

$$\text{s.t.} \sum_{i\in\mathcal{V}:(i,j)\in\mathcal{A}} x_{ij} = 1, \qquad j \in \mathcal{C} \tag{2}$$

$$\sum_{j\in\mathcal{V}:(i,j)\in\mathcal{A}} x_{ij} - \sum_{j\in\mathcal{V}:(j,i)\in\mathcal{A}} x_{ji} = 0, \qquad i \in \mathcal{C} \tag{3}$$

$$\sum_{j\in\mathcal{V}:(o,j)\in\mathcal{A}} x_{oi} - \sum_{i\in\mathcal{V}:(i,d)\in\mathcal{A}} x_{id} = 0 \tag{4}$$

$$w_j \geq w_i + q_j x_{ij} - Q(1 - x_{ij}), \qquad (i,j) \in \mathcal{A} \tag{5}$$

$$q_i \leq w_i \leq Q, \qquad i \in \mathcal{V} \tag{6}$$

$$y_j(\delta) \geq y_i(\delta) + t_{ij}(y_i(\delta), \delta) + T_{ij}(1 - x_{ij}), \qquad (i,j) \in \mathcal{A}, \delta \in \Delta_\Gamma \tag{7}$$

$$e_i \leq y_i(\delta) \leq l_i, \qquad i \in \mathcal{V}, \delta \in \Delta_\Gamma \tag{8}$$

$$x_{ij} \in \{0,1\}, \qquad (i,j) \in \mathcal{A}. \tag{9}$$

Inequalities (7) and (8) encapsulate two key challenges of the problem: (i) the *recourse variable* $y_i(\delta)$ that depends on a specific scenario; and (ii) the time-dependent function that depends both on the recourse variable and on the uncertain vector δ. Due to these inequalities, the formulation is nonlinear and not compact, thus it is not clear that the decision version of the problems lies in $\mathcal{NP}$.

3 Determining feasibility

In what follows, we assume that x satisfies all routing, flow, and capacity constraints. Thus, the vector x decomposes into a set of at most $|\mathcal{C}|$ paths from o to d. Let π be any of these paths. To simplify the notation, we consider π as a sequence of arbitrary vertices of $\mathcal{V}$ ordered as $1, 2, \ldots, n$, where $n = |\pi|$. In addition, we denote $t_{ij}(s, \delta)$ as $t_i(s, \delta)$ to indicate the travel time from $i \in [n-1]$ to its subsequent vertex in π at $s \in \mathbb{R}_+$ for a scenario $\delta \in \Delta_\Gamma$, where $[\ell] = \{1, \ldots, \ell\}$ (and $[\ell]_0 = [\ell] \cup \{0\}$) for any positive integer ℓ. Let $a_i(\delta)$ be the arrival time at $i \in [n]$ in a scenario $\delta \in \Delta_\Gamma$, which is recursively defined as follows:

$$a_i(\delta) = \begin{cases} e_1 & \text{if } i = 1, \\ \max\{e_i,\, a_{i-1}(\delta) + t_{i-1}(a_{i-1}(\delta), \delta)\} & \text{otherwise.} \end{cases} \tag{10}$$

The time-window infeasibility of π occurs if any $a_i(\delta) > l_i$ for $i \in [n]$ and $\delta \in \Delta_\Gamma$. Hence, checking whether x satisfies the time-window constraints reduces to verifying the following feasibility problem for eahc path π induced by x:

ROBUST TIME-DEPENDENT PATH FEASIBILITY PROBLEM (RTDPFP)
INSTANCE: A path $\pi = (1, 2, \ldots, n)$ such that each vertex has a time window $[e_i, l_i] \subseteq [0, H]$ for $i \in [n]$ and each arc on the path is a time-dependent travel time function $t_i(s, \delta) = \bar{t}_i(s) + \delta_i \hat{t}_i(s)$, for $s \in \mathbb{R}_+$, where $\bar{t}_i(s)$ and $\hat{t}_i(s)$ are piecewise linear functions with p breakpoints.
QUESTION: Is there a scenario $\delta \in \Delta_\Gamma$ and $i \in [n]$, such that $a_i(\delta) > l_i$?

Theorem 3.1. *If Assumption 1 is relaxed, the RTDPFP is $\mathcal{NP}$-complete.*

Proof. The proof is omitted due to space restrictions. □

The case when Assumption 1 is relaxed is unrealistic because it does not respect the effect of *non-passing*. Typical models for the TDVRP consider the FIFO property, formally in Assumption 1, which means that if a vehicle leaves from a vertex i to a vertex j at a given time, leaving vertex i at a later time implies arriving later at vertex j.

When the FIFO property holds, we can extend the DP approach presented by Agra et al. (2013) to our setting. This is because arriving later at a given node is always detrimental with respect to the satisfaction of the upper time window constraints. Hence, the purpose of the adversary is to delay the arrival times as much as possible at all nodes of the path. Specifically, let $\alpha(i, \gamma)$ indicate the maximum arrival time at vertex $i \in [n]$ for a given budget $\gamma \in [\Gamma]_0$. We compute each state as follows:

$$\alpha(i,\gamma) = \begin{cases} e_1 & \text{if } i = 1, \\ \max\{e_i, \alpha(i-1,\gamma) + \bar{t}_{i-1}(\alpha(i-1,\gamma))\} & \text{if } i > 1 \textbf{ and } \gamma = 0, \\ \max\{e_i, \alpha(i-1,\gamma) + \bar{t}_{i-1}(\alpha(i-1,\gamma)), & \\ \quad \alpha(i-1,\gamma-1) + \bar{t}_{i-1}(\alpha(i-1,\gamma-1)) + \hat{t}_{i-1}(\alpha(i-1,\gamma-1))\} & \text{otherwise.} \end{cases} \tag{11}$$

The path is feasible if $\alpha(i, \Gamma) \leq l_i$ for every $i \in [n]$. A binary search is sufficient to obtain the values of $\bar{t}_{i-1}$ and $\hat{t}_{i-1}$ from $\bar{B}_{ij}$ and $\hat{B}_{ij}$. Thus, we obtain the following result:

Proposition 1. *When Assumption 1 holds, DP recurrence* (11) *solves the RTDPFP in* $\mathcal{O}(n\Gamma \log P)$.

4 Exact approaches for the RTDVRPTW

4.1 Compact formulation (CF)

The CF is derived by reformulations of the constraints (7) and (8). The first step to obtain a CF comes with the linearization of recursion (11) similarly as in Munari et al. (2019). Let α_i^γ

be a nonnegative real variable that represents each state $\alpha(i,\gamma)$ of the recursion (11). We can then write the following inequalities:

$$\alpha_i^\gamma \geq e_i, \qquad i \in \mathcal{V}, \gamma \in [\Gamma]_0 \tag{12}$$
$$\alpha_j^\gamma \geq \alpha_i^\gamma + \bar{t}_{ij}(\alpha_i^\gamma) - \bar{T}_{ij}(1 - x_{ij}), \qquad (i,j) \in \mathcal{A}, \gamma \in [\Gamma]_0 \tag{13}$$
$$\alpha_j^\gamma \geq \alpha_i^{\gamma-1} + \bar{t}_{ij}(\alpha_i^{\gamma-1}) + \hat{t}_{ij}(\alpha_i^{\gamma-1}) - \hat{T}_{ij}(1 - x_{ij}), \qquad (i,j) \in \mathcal{A}, \gamma \in [\Gamma], \tag{14}$$

where $\bar{T}_{ij} = l_i + \bar{t}_{ij}(l_i)$ and $\hat{T}_{ij} = \bar{T}_{ij} + \hat{t}_{ij}(l_i)$. Following Munari et al. (2019), we see that $\alpha(i,\Gamma) \leq l_i$ if and only if there exists a solution to (12)–(14) that also satisfies

$$\alpha_i^\Gamma \leq l_i \qquad i \in \mathcal{V}. \tag{15}$$

Observe that inequalities (13) and (14) are nonlinear because of the piecewise affine functions $\bar{t}_{ij}$ and $\hat{t}_{ij}$. We detail below how to handle them using SOS2 constraints and the sets of breakpoints $\bar{B}_{ij}$ and $\hat{B}_{ij}$. Let $\bar{z}_{ij}^{p\gamma}$ (resp. $\hat{z}_{ij}^{p\gamma}$) for $p \in [P]$ be nonnegative real variables that form an SOS2 and sum up to 1, for each $(i,j) \in \mathcal{A}$, $\gamma \in [\Gamma]$. Then, observing that functions $\bar{t}_{ij}$ and $\hat{t}_{ij}$ need to be linearized only when $x_{ij} = 1$, we have

$$x_{ij} = 1 \implies \begin{cases} (\alpha_i^\gamma, \bar{t}_{ij}(\alpha_i^\gamma)) = \sum_{p\in[P]} \bar{z}_{ij}^{p\gamma}(\bar{\mathfrak{s}}_{ij}^p, \bar{\mathfrak{t}}_{ij}^p) \\ (\alpha_i^\gamma, \hat{t}_{ij}(\alpha_i^\gamma)) = \sum_{p\in[P]} \hat{z}_{ij}^{p\gamma}(\hat{\mathfrak{s}}_{ij}^p, \hat{\mathfrak{t}}_{ij}^p) \end{cases}, (i,j) \in \mathcal{A}, \gamma \in [\Gamma]_0. \tag{16}$$

Thus, we can reformulate (13) and (14) as follows:

$$\sum_{p\in[P]} \bar{\mathfrak{s}}_{ij}^p \bar{z}_{ij}^{p\gamma} - \bar{T}_{ij}(1-x_{ij}) \leq \alpha_i^\gamma \leq \sum_{p\in[P]} \bar{\mathfrak{s}}_{ij}^p \bar{z}_{ij}^{p\gamma} + \bar{T}_{ij}(1-x_{ij}), \qquad (i,j) \in \mathcal{A}, \gamma \in [\Gamma]_0 \tag{17}$$
$$\sum_{p\in[P]} \hat{\mathfrak{s}}_{ij}^p \hat{z}_{ij}^{p\gamma} - \hat{T}_{ij}(1-x_{ij}) \leq \alpha_i^\gamma \leq \sum_{p\in[P]} \hat{\mathfrak{s}}_{ij}^p \hat{z}_{ij}^{p\gamma} + \hat{T}_{ij}(1-x_{ij}), \qquad (i,j) \in \mathcal{A}, \gamma \in [\Gamma] \tag{18}$$
$$\alpha_j^\gamma \geq \sum_{p\in[P]} (\bar{\mathfrak{s}}_{ij}^p + \bar{\mathfrak{t}}_{ij}^p)\bar{z}_{ij}^{p\gamma} - \bar{T}_{ij}(1-x_{ij}), \qquad (i,j) \in \mathcal{A}, \gamma \in [\Gamma]_0 \tag{19}$$
$$\alpha_j^\gamma \geq \sum_{p\in[P]} [(\bar{\mathfrak{s}}_{ij}^p + \bar{\mathfrak{t}}_{ij}^p)\bar{z}_{ij}^{p,\gamma-1} + \hat{\mathfrak{t}}_{ij}^p \hat{z}_{ij}^{p,\gamma-1}] - \hat{T}_{ij}(1-x_{ij}), \qquad (i,j) \in \mathcal{A}, \gamma \in [\Gamma] \tag{20}$$
$$\{\bar{z}_{ij}^{0\gamma}, \ldots, \bar{z}_{ij}^{P\gamma}\} \in \text{SOS2}, \qquad (i,j) \in \mathcal{A}, \gamma \in [\Gamma]_0 \tag{21}$$
$$\{\hat{z}_{ij}^{0\gamma}, \ldots, \hat{z}_{ij}^{P\gamma}\} \in \text{SOS2}, \qquad (i,j) \in \mathcal{A}, \gamma \in [\Gamma] \tag{22}$$
$$\sum_{p\in[P]} \bar{z}_{ij}^{p\gamma} = 1, \qquad (i,j) \in \mathcal{A}, \gamma \in [\Gamma]_0 \tag{23}$$
$$\sum_{p\in[P]} \hat{z}_{ij}^{p\gamma} = 1, \qquad (i,j) \in \mathcal{A}, \gamma \in [\Gamma], \tag{24}$$
$$\bar{z}, \hat{z} \geq 0, \tag{25}$$

where (17) and (18) are explicit linearizations of the first equalities in (16), while both equalities of (16) have been plugged into the right-hand sides of (13) and (14), leading to (19) and (20).

Remark 4.1. *The decision version of the RTDVRPTW is in $\mathcal{NP}$.*

4.2 Scenario generation algorithm (SGA)

Given that only a limited number of scenarios are typically necessary for the model to reach the optimal solution, the principle of the SGA (Zeng and Zhao, 2013) is to iteratively solve a relaxed and tractable formulation and, for each solution obtained, add scenarios that yield the incumbent solution infeasible, repeating this process until no such scenarios can be found.

The relaxed formulation depends on a restricted set of scenarios $\tilde{\Delta}_\Gamma \subseteq \Delta_\Gamma$. The MILP includes variables y_i^δ and constraints (7) and (8), for $\delta \in \tilde{\Delta}_\Gamma$. Once again, we can reformulate the time-dependent functions using SOS2 constraints. For every solution x' obtained from the formulation of $\tilde{\Delta}_\Gamma$ the separation problem finds a set of scenarios that yields an infeasible x' regarding the robust time windows. This set of scenarios is computed by means of (11).

4.3 B&C with lazy separation of tournament inequalities (B&C-TI)

Our last formulation handles the robust time window constraints using simple extensions of the tournament inequalities (TI) proposed in Ascheuer et al. (2000).

The separation problem involves identifying infeasible paths. We decompose the linear relaxation of x from flow values into paths employing a depth-first-search algorithm. If the minimum flow of the constructed path is greater than a threshold, we address (11) to check the feasibility. We add these inequalities in a lazy fashion in the course of a B&C algorithm.

4.4 Algorithmic enhancements

We include inequalities *rounded capacity inequalities* (RCIs) in a cutting plane fashion, since they are known to improve the lower bound of the linear relaxation in several VRPs. For the separation problem, we used the `CVRPSEP` package (Lysgaard, 2003) to identify a violated subset of customers, based on a given fractional solution of x.

Furthermore, we propose three preprocessing procedures that can be used to enhance the performance of the approaches when solving RTDVRPTW. The first two are extensions from classical time-window tightening and arc removal, both traditionally applied when solving VRPTWs. The third one relies on reducing the number of breakpoints for each arc.

Finally, we implemented a multi-start ILS algorithm to provide strong initial primal bounds for the exact approaches and to produce high-quality solutions for instances of realistic size. The method follows the standard ILS procedure (Subramanian and Lourenço, 2022), executed independently I_{Max} times, returning the best solution across all restarts. Each restart performs up toI_{ILS} consecutive iterations without improvements to the incumbent of that restart.

5 Computational experiments

We considered two sets of instances in our numerical experiments. The first relies on adapted classical instances from the literature to examine the solution methods. The second relies on real data from a transportation service company operating in Paris, which is used to construct the uncertainty sets and to perform simulations for evaluating different models.

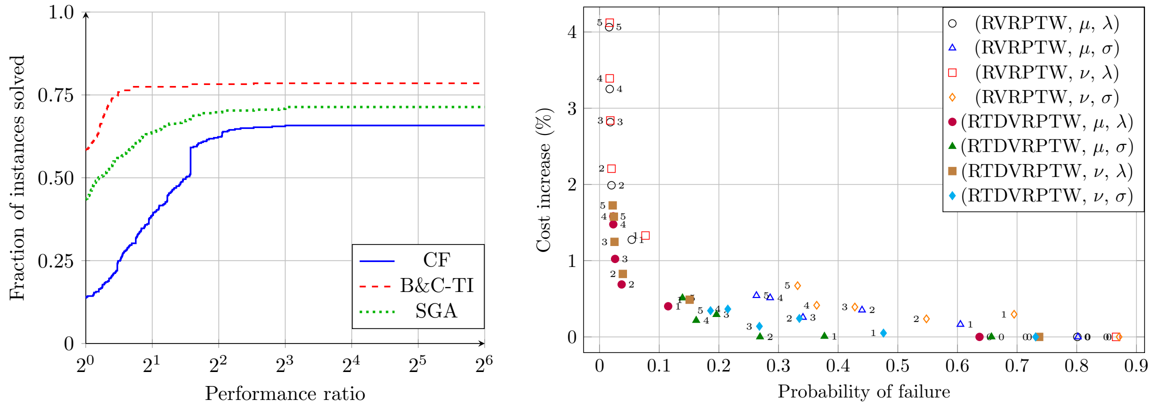
We first present the results achieved by the three proposed exact approaches described in Section 4, that were strengthened by algorithmic enhancements discussed in Section 4.4 over the an adaptation of the instances of Dabia et al. (2024) with $|\mathcal{C}| = 25$.

Figure 1a shows the performance profiles of the three exact approaches. The x-axis represents the factor by which a method's CPU time exceeds that of the best algorithm for each instance, and the y-axis gives the fraction of instances solved within that ratio. B&C-TI clearly dominates, solving about 78% of the instances within at most six times the CPU time of the best method, whereas SGA and CF solved around 70% and 60% of the instances, respectively. Note that the curves for B&C-TI and SGA plateaued earlier, indicating that most of their solvable instances required no more than twice the best observed time, while CF required comparatively higher performance ratios to reach its final fraction of solved instances.

Our second experiments reports computational results on instances derived from real-world data for the VRPTW, TDVRPTW, RVRPTW, and RTDVRPTW models. The goal is to empirically validate the superior performance of the proposed RTDVRPTW model through simulation-based comparisons. Since the instances are larger ($|\mathcal{C}| = 150$), we focused exclusively on the solutions attained by the ILS heuristic, as the exact methods already faced difficulties when solving instances with $|\mathcal{C}| = 25$. The key idea to the experimentation is to split the data into *in-sample* data to construct the uncertainty sets used as input data for the algorithm and *out-of-sample* data to evaluate it by performing simulations in a cross-validation fashion.

Our uncertainty sets are based on four statistical functions. Specifically, the mean (μ) and median (ν) for the nominal values, and maximum value (λ) and the standard deviation (σ) for the deviation values.These definitions allow us to design four combinations of uncertainty sets, (μ, σ), (μ, λ), (ν, σ), and (ν, λ) for each model.

Figure 1b compares the methods simultaneously in terms of both cost increase and probability of failure, including all configurations of uncertainty sets and models. Next to each point on the chart, the corresponding Γ value is indicated. The non-dominated configurations appear in the lower-left region of the plot, primarily represented by the RTDVRPTW models configured with the mean as the nominal value (μ), and either the maximum as the deviation (λ) with $\Gamma \in \{1, 2\}$, or the standard deviation as the deviated value (σ) with $\Gamma \in \{2, 4\}$.



(a) Performance profile

(b) Price of robustness

FIG. 1: Computational results of the RTDVRPTW

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