Empty containers return optimization problem

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Introduction

Every day, hundreds of trucks travel across France to deliver mail to its recipients. These items are transported in three types of standardized containers, ensuring consistent and efficient processing across all *Industrial Mail Platforms* (IMP). Since the number of containers is limited and mail flows are asymmetrical, a balancing process is required to prevent container shortages within the IMP network. Every day, each site therefore reports its needs or surplus of containers by type and quantity. The balancing process must then minimize the number of additional trucks by using as much of the remaining space as possible in the trucks already delivering mail.

As quantum computing has advanced in recent years, companies such as La Poste, which face large-scale logistical challenges, are becoming increasingly interest in this technology. Problems like the one presented in the previous paragraph can serve as an ideal entry point to assess quantum capabilities and explore methods for modeling and solving such operational problems. Beyond this specific case, La Poste's broader objective is to prepare for future technological shifts. This involves first verifying that its logistical challenges can indeed be modeled within the emerging quantum programming paradigms, and then developing formulations that are compatible with current and upcoming hardware. By doing so, La Poste aims to be ready to leverage quantum computing as soon as the technology reaches maturity.

In this paper, we propose a quantum-hybrid approach to solve this problem using intermediate-scale quantum computers.

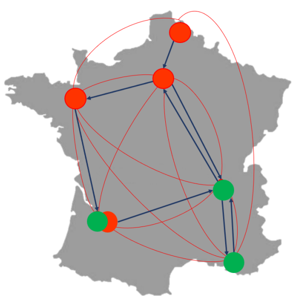

FIG. 1: A toy example of the problem: existing mail truck routes in blue, and possible additional trucks in red.

Modeling

The problem described in the introduction can be formulated as a three-commodity flow problem, with the particularity that all commodities can share the same transport channel and that the cost function depends not on the quantity transported, but on the number of additional trucks used, which makes the function nonlinear. In addition, each truck has a fixed minimum cost. Furthermore, existing trucks operate along transport lines that are already scheduled over time. This means that one cannot switch from one line to another without taking time constraints into account.

We model the problem as follows :

Let V denote the set of IMPs and let $L_i = \{(v_{i1}, v_{i2}), \dots, (v_{in_i-1}, v_{in_i})\}$, (v_{ik}, v_{ik+1}) be the existing transport lines. Let $G = (S, R, V_L, E_L, E_e)$ where

- $S \subseteq V$ is the set of supplier sites,
- $R \subseteq V$ is the set of requesting sites,
- $V_L = \{v_{ij} | \ \forall i, \forall j\}$ is the set of logical nodes derived from existing lines,
- $E_L = \{\bigcup_i L_i\}$ is the set of arcs corresponding to existing lines,
- $E_e = \left\{ \left((S \times (V_L \cup R)) \cup V_L \times R \right) \setminus E_L \right\}$ is the set of arcs corresponding to additional lines.

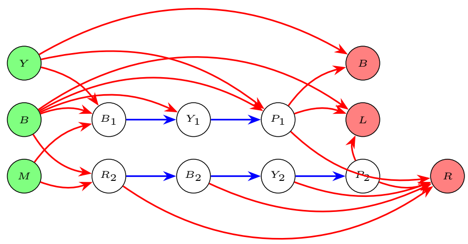

FIG. 2: The graph G (not all edges are included)

$$
\begin{aligned}
\min \quad & \sum_{(u,v)\in E} \sum_{k\in K} c^k_{uv} y^k_{uv} & & \\
\text{s.t.} \quad & \sum_{k\in K} x^k_{uv} \leq \beta_{uv} & & \forall (u,v) \in E \quad \text{(Capacity)} \\
& \sum_{u:(u,v)\in E} x^k_{uv} - \sum_{w:(v,w)\in E} x^k_{vw} = S^k_v & & \forall v \in V, \ \forall k \in K \quad \text{(Truck Flow Balance)} \\
& y^k_{uv} - 1 \leq \frac{x^k_{uv}}{Q} \leq y^k_{uv} & & \forall (u,v) \in E, \ \forall k \in K \quad \text{(Ceiling)} \\
& x^k_{uv} \in \mathbb{N} & & \forall (u,v) \in E, \ \forall k \in K \quad \text{(Integrality)}
\end{aligned}
$$

Solving our problem require to determine a set of shortest paths from supplier sites that meet the demand requirements of the requesting sites.

Algorithm

In order to use a quantum computer to solve this problem, one could attempt to implement the corresponding QUBO [1]. However, since the problem involves integer variables and inequality constraints, it would require too many variables to be practical.

Inspired by Ford Fulkerson algorithm, we propose the following two-step procedure:

1. Compute an assignment of quantities for direct deliveries from supplier sites to requesting sites without using E_L.
2. Update the set of paths that satisfy the assignment from step 1 while maximizing the use of free travel provided by E_L.

The second step can be executed using a quantum shortest-path algorithm [2].

References

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