A Methodology for Simulating High Depth QAOA

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Mots-clés : *Tensor Network, QAOA, Shallow Quantum Circuit*

1 Introduction

The Quantum Approximate Optimization Algorithm (QAOA) has become one of the most extensively investigated variational quantum algorithms for solving combinatorial optimization problems[1]. However, evaluating the behavior of QAOA for large depth Quantum Circuit (QC) or on large scale optimization instances remains challenging. Noisy quantum hardware cannot reliably support deep circuits with many two-qubit gates, and classical state vector simulation scales exponentially with the number of qubits which restraints direct numerical studies to relatively very small problems.

Tensor Networks (TN) and specifically Matrix Product States (MPS) provide a more scalable alternative for simulating certain QAOA by representing quantum states using local tensors connected through bond dimensions that capture entanglement. Classical MPS based QAOA simulation can efficiently handle early QAOA layers, especially on sparse or low dimensional interaction graphs [2]. However, as the circuit depth p grows, the induced entanglement typically increases and eventually requires a bond dimension that becomes computationally expensive. This bond dimension barrier limits the depth to which classical MPS simulation can be applied.

Recent work on variational preparation of normal MPS, called Approximate Quantum Computing (AQC), shows that MPS descriptions can be compiled into hardware efficient shallower circuits [3], so that the same target state can be realized on quantum hardware with much lower circuit depth. Motivated by these developments, we explore a methodology that combines classical MPS simulation with shallow circuit compilation in the context of QAOA optimization to evaluate how these recent trends could help pushing the boundaries of QAOA experimentation.

2 Methodology

The proposed method follows a hybrid workflow that should extend the practical depth of QAOA, as is shown in the Figure 1. The central idea is that the early part of the QAOA circuit is simulated classically using a MPS, while being approximated through AQC to a shallower equivalent quantum circuit. Once concatenated the approximated circuit and the later layers are then executed on quantum hardware to produce the QAOA quantum state ready for evaluation. As shown in the figure, and following the general scheme of AQC provided in [3], we first divide the full QAOA circuit into two parts. For a chosen prefix depth p_0, the first p_0 layers form a block that we denote U_1. Only this block will be simulated classically. The rest of the circuit, denoted U_2, is intended to run directly on hardware.

The prefix block U_1 is applied to the initial state using an MPS representation. Mixer gates act on single sites of the tensor network. Cost gates act on pairs of neighboring sites and are implemented by combining the two tensors, applying the gate, and splitting the tensor again using a singular value decomposition. During this process, the bond dimension of the MPS can be monitored. A fixed limit or truncation rule controls how much entanglement can be kept. Classical simulation continues only while the bond dimension stays within this limit. When

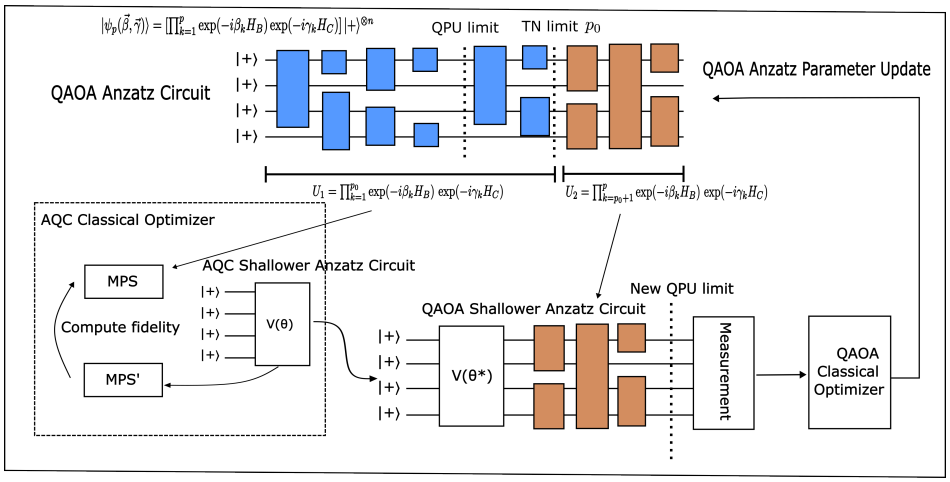

FIG. 1 – Shallow parameterized circuit for QAOA

the next QAOA layer would exceed the limit, the current circuit depth p_0 could be seen as the largest feasible depth for classical simulation. All these steps can be achieved with a standard quantum simulator such as qiskit MPS simulator.

The intermediate state produced at depth p_0 is then used as the target for Approximate Quantum Compiling. A shallow parameterized circuit $V(\theta)$ is optimized trough its parameters so that it prepares the same state as the intermediate MPS. The optimization uses tensor network contractions to compute fidelity and parameter updates are provided by a classical optimizer, therefore the entire compilation process is classical and does not depend on quantum hardware. A fixed brickwork circuit can be used as in AQC-Tensor, or an adaptive structure can be built as in ADAPT-AQC [3]. Both approaches should produce a shallow circuit $V(\theta^*)$ that prepares the intermediate state with high fidelity. This shallow circuit replaces the entire prefix block U_1 such that both U_1 and U_2 could run on the quantum device.

3 Conclusion and Perspectives

We propose to leverage the recent advances in hybrid tensor-quantum computing in the context of QAOA. Since the interchangeability between QC and MPS models, we expect that preparing product states variationally should help analyze the structure and limitations of QAOA more deeply. The method offers a systematic Tensor Network based view of QAOA, helping identify improvements via bond dimension and entanglement control or through deeper circuits. Future work includes applying this framework to concrete problem instances, validating its predictions on small scale quantum hardware or simulators, as a special case of the many powerful potentials of AQC.

References

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