Reliability-Based Measure of a Retrial Machine Repair System Involving Imperfect Coverage and Preventive Maintenance

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1 Introduction

Maintaining a high level of reliability is a fundamental prerequisite for industries seeking to enhance production efficiency and reduce operational costs. Retrial queues (or Retrial Machine Repair Systems, RMRS) have been extensively adopted as mathematical models in various domains, such as communication networks, data centers, and cluster computing[1]. While production equipment often employs real-time monitoring, failure of the detection mechanism results in *imperfect coverage*, causing the system to enter an unsafe state (denoted by u_f) that requires manual restoration[2]. When a machine fails, repair activities are carried out by a repair facility. To enhance system reliability, the repair facility may adopt *preventive maintenance* (PM); specifically, when idle, it performs inspections or replaces components to ensure that the repair facility remains in optimal operating condition. Introducing standby redundancy and adopting PM at the repair facility are effective approaches to improving reliability. This study proposes a novel RMRS that simultaneously integrates warm standbys, imperfect coverage, and PM. The model serves as an analytical framework for network and cluster environments, facilitating the evaluation of key system parameters, such as failure, repair, and PM rates. Consequently, the findings provide theoretical insights and practical guidelines for optimizing system reliability and management strategies.

2 Description of the RMRS

This study investigates a single-server RMRS subject to failures and maintenance activity. This system consists of $L = M + S$ identical components, where M denotes the number of primary components and S denotes the number of warm standby components. The times to failure for primary and warm standby components follow exponential distributions with failure rates λ and α $(0 < \alpha < \lambda)$, respectively. The probability of successful detection, location, and recovery on the failure of a primary or standby component is denoted by q. If a failure occurs but is not successfully covered, a reboot operation is required to clear the u_f state. The reboot delay times are assumed to be exponentially distributed with parameter β. The server can repair one failed component at a time, and there is no waiting area for failed component in the system. The repair times are assumed to be exponentially distributed with parameter μ. If a failed component finds the server idle, it is repaired immediately. Otherwise, it enters a retrial orbit. If a failed component finds the server idle, it is repaired immediately. Otherwise, it enters a retrial orbit. The failed component waits in the retrial orbit and retries to request maintenance services after waiting a random period of time. The successful retrial times follow an exponential distribution with parameter γ. PM of the server occurs according to a Poisson process with rate η. We assume that the PM durations follow an exponential distribution with parameter δ. Finally, failure times, repair times, PM times, retrial times, and reboot delay times are all independent of each other.

3 System reliability and mean time to system failure

In the proposed RMRS with imperfect coverage and PM, the system state at time t is represented by the pair (i, j). Let i denote the status of the server, where $i = 0$ indicates the server is idle, $i = 1$ indicates the server is busy, and $i = 2$ indicates the server is undergoing PM. Furthermore, let $j = n$ represent that there are n failed components in orbit when the system is in safe failure state, and $j = uf_n$ represent that there are n failed components in orbit when the system is in u_f state. Let $P_{i,j}(t)$ be defined as the probability that the system is in state (i, j) at time t. Based on these definitions and assumptions, we first construct the state–transition rate diagram. Using this diagram, we derive the differential–difference equations of the retrial system and subsequently transform them into their Laplace forms. A matrix-analytic method is then employed to obtain the steady-state probability solutions. Finally, the system reliability $R_Y(t)$ and the mean time to system failure $(MTTF)$ are defined as $R_Y(t) = 1 - P_{2,L}(t) - P_{1,L-1}(t)$ and $MTTF = \int_0^\infty R_Y(t)\,\mathrm{d}t$.

4 Numerical results

To examine the effects of system parameters such as λ, μ, γ, δ and q on the $R_Y(t)$ and $MTTF$, we adopt $M = 12$, $S = 4$, $\lambda = 0.12$, $\alpha = 0.2\lambda$, $\mu = 1.6$, $\gamma = 0.6$, $q = 0.8$, $\beta = 0.8$, $\eta = 1.0$, and $\delta = 0.3$ as the baseline case. All computations were executed using the Maple symbolic computation system.

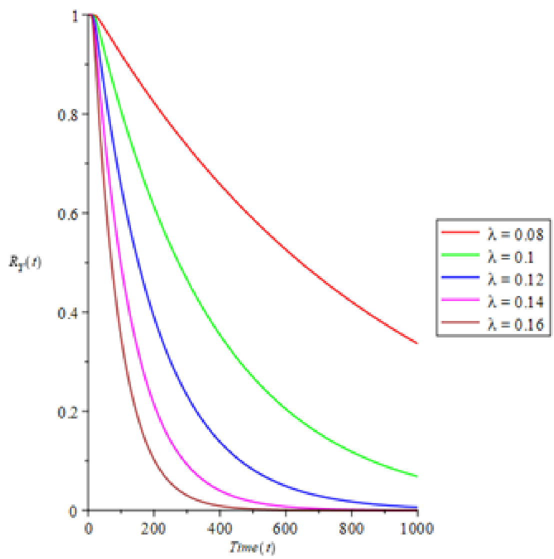

FIG. 1: System reliabilty affected by λ

TAB. 1: $MTTF$ affected by λ and μ

λ \ μ	1.2	1.4	1.6	2.0	2.4
0.08	834.603	899.222	947.726	1015.120	1059.420
0.10	372.139	395.992	413.432	436.905	451.824
0.12	218.470	230.142	238.414	249.075	255.507
0.14	150.572	157.416	162.115	167.856	171.063
0.16	114.618	119.166	122.206	125.709	127.472

From FIG 1, we observe that λ significantly affect system reliability. Furthermore, μ, γ and δ affect the system reliability moderately and q affects the system reliability slightly from other figures. It reveals from TAB 1 that $MTTF$ decreases as λ increases. On the other hand, $MTTF$ increases as μ increases. These numerical results suggest that investing in higher-quality equipment to reduce failure rates is an effective strategy for enhancing system reliability.

References

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