

Computational complexity of radio-astronomical observation scheduling

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1 Introduction

This work tackles the scheduling of radio-astronomical observations, in the context of the Square-Kilometre Array Observatory project (SKAO) and in partnership with the ECLAT laboratory. As an observer from the Earth, one sees celestial bodies move around in the sky over time. The main factor being the Earth revolving around itself, plotting the angle of altitude of a celestial body over time yields a near-sinusoidal signal on a twenty-four hour period. In order to mitigate the impact of the atmosphere of Earth on observation quality, a global altitude lower bound is set - typically 20° , at least above the horizon line (0°). Setting such an altitude threshold leads to a daily availability window for each celestial body, which could overlap part of two consecutive days.

Consequently, this setting can be modeled as the makespan minimization of a single machine scheduling non-preemptive instance with a set of n jobs $\mathcal{J} = \{1, \dots, n\}$, a day length $L \in \mathbb{R}_{>0}$ and, for each job j in \mathcal{J} , a processing time $p_j \in \mathbb{R}_{>0}$, a daily release date $r_j \in [0, L)$ and a daily deadline $d_j \in [0, L)$. We denote this problem by $1|periodic_L(r_j, d_j)|C_{max}$. Surprisingly, to the best of our knowledge, this does not seem to correspond to any previously studied scheduling problem. As such, in this contribution, we propose to investigate the (parameterized) complexity of this scheduling problem, and unveil connections to more established scheduling settings.

2 Contribution and related problems

First, we consider the computation of a schedule of minimum makespan OPT . According to the single machine setting with regular time windows $1|r_j, d_j|C_{max}$, our problem is (strongly) NP-complete even on a single day [5]. It even generalizes machine minimization problem $P|r_j, d_j|\min(m)$, where each machine is interpreted as a day. In response, as a baseline, we propose a dynamic programming algorithm computing the minimum makespan over all job subsets in $\mathcal{O}(n \cdot 2^n)$ time, where jobs are added to the schedule one by one. We also show that the preemptive variant of our problem is polynomial-time solvable by providing a translation into a maximum flow problem with $\mathcal{O}(n)$ nodes and $\mathcal{O}(n^2)$ edges.

Note that our setting is reminiscent of the discrete Interval Scheduling problem, where a collection of arbitrary start times (i.e., not periodic and not necessarily consecutive) is given to each job. While scheduling equal-length jobs with three arbitrary start times per job is (strongly) NP-complete [4], our setting can be solved in $\mathcal{O}(n \cdot \log(n)^2)$ time when time windows are tight - i.e., when their length is equal to the job processing time [2]. This highlights the periodicity of job availabilities as a key property in our setting.

Regarding approximation, we took inspiration from the machine minimization setting [1] to propose a $\mathcal{O}(\log(n))$ -approximation algorithm whenever $OPT \geq L$. This is based on a $1/2$ -approximation algorithm on the Job Interval Selection problem, where the goal is to maximize the number of scheduled jobs [6]. We also show that, unless $P = NP$, there is no $(2 - \varepsilon)$ -approximation algorithm.

Finally, regarding fixed-parameter tractability, we consider four parameters: the number of days $\#days$, the width μ - i.e., the maximum number of overlapping time windows -, the slack σ - i.e., the maximum difference between the length of the daily time window of a job and its processing time - and the border flexibility β - i.e., the maximum, over all jobs j , of the minimum between their processing time minus one and their slack plus one. Our problem is para-NP-complete parameterized by each individual parameter [1, 3]. We show that our problem is fixed-parameter tractable parameterized by $(\#days + \sigma)$ and $(\mu + \beta)$ by adapting existing fixed-parameter algorithms on the identical parallel machine setting, respectively based on start time enumeration [1] and border schedule enumeration [7]. In the former we enumerate over days on top of daily start times, while in the latter we enumerate over day-overlapping schedules, then have the rest of the algorithm unfold in a similar way.

3 Discussion

This work gives a first insight on the computational complexity of radio-astronomical observation scheduling. Theoretically speaking, we believe that the main remaining open question is whether there is a constant approximation algorithm for our problem. In practice, current fixed-parameter algorithms are too slow to be used in practice, even compared to the baseline exact algorithm. Going forward, we believe that the model could be refined accordingly to the real-life setting. For instance, radio-observations are geared towards gathering low-frequency signals and thus often take several hours to complete, while switching the target only takes a couple minutes. So the time span could be discretized with a fairly large time step (e.g., 15 mn), and L could be considered to be given in unary in the input. Furthermore, observations can usually be split in chunks, as long as the length of each chunk is long enough. Thus some limited preemption could be introduced to the model in order to decrease the minimum makespan value significantly.

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