Identifying Critical Infrastructure: A Bilevel Genetic Algorithm for the Facility Interdiction Problem

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1 Introduction

Modern infrastructure systems, such as supply chains, transportation networks, and communication grids, are inherently vulnerable to disruptions from targeted attacks, technical failures, or natural hazards. The loss of key facilities can cascade into increased operational costs, service degradation, and systemic inefficiency. From a defensive viewpoint, system operators must anticipate worst-case disruption scenarios to prioritize protective measures and allocate hardening resources effectively. This challenge is formalized as the *Facility Interdiction Problem* (FIP) [1].

The FIP models a strategic attacker with the capability to destroy exactly k facilities, aiming to maximize post-disruption service cost. The defender's goal is to identify which k facilities, if lost, would cause the greatest damage, thus revealing the most critical assets for protection. While classical facility location models focus on optimal placement, interdiction models introduce an adversarial perspective essential for vulnerability assessment and resilience planning.

This work presents a clear bilevel formulation of the FIP and an efficient metaheuristic solution based on a Genetic Algorithm (GA). Our approach explicitly captures the sequential attacker–defender interaction and provides a scalable computational tool for identifying critical infrastructure components.

2 Problem Definition and Mathematical Model

We consider a system with a set of demand nodes $I = \{1, \ldots, n\}$ and a set of facilities $J = \{1, \ldots, p\}$. Each demand node i has an associated demand rate h_i, and c_{ij} denotes the service cost (e.g., travel distance) between node i and facility j. An adversary can interdict exactly k facilities.

The problem is modeled as a Stackelberg (bilevel) game in which an upper-level attacker chooses a subset of k facilities to interdict so as to *maximize* the total system cost induced after re-assignment, while a lower-level defender, observing this interdiction, assigns each demand node to an operational facility in order to *minimize* the resulting total weighted service cost.

Let y_j be a binary variable indicating whether facility j remains operational ($y_j = 1$) or is interdicted ($y_j = 0$). Let x_{ij} indicate whether demand node i is assigned to facility j after interdiction.

The attacker's problem is:

$$\max_{y} Z(y) \tag{1}$$

$$\text{s.t.} \sum_{j \in J} y_j = p - k \tag{2}$$

$$y_j \in \{0, 1\}, \quad \forall j \in J$$

where $Z(y)$ is the optimal value of the lower-level problem:

$$\begin{aligned} Z(y) = \min_{x} \sum_{i \in I} \sum_{j \in J} h_i c_{ij} x_{ij} && (3) \\ \text{s.t.} \sum_{j \in J} x_{ij} = 1, \quad \forall i \in I && (4) \\ x_{ij} \le y_j, \quad \forall i \in I, \forall j \in J && (5) \\ x_{ij} \in \{0,1\}, \quad \forall i \in I, \forall j \in J && \end{aligned}$$

Constraint (2) ensures exactly k facilities are interdicted. In the lower level, (4) requires each demand node to be assigned to exactly one facility, and (5) permits assignment only to operational facilities.

3 Proposed Solution

To solve the bilevel interdiction model, we employ a GA to efficiently explore the combinatorial space of possible facility interdictions. In this approach, solutions are represented as vectors indicating which k facilities to remove. The GA evaluates candidate interdiction sets by solving the corresponding lower-level assignment subproblem, which determines the optimal system response to a given set of disabled facilities. The total cost resulting from this optimal reassignment serves as the fitness value, driving the evolutionary search toward the most disruptive interdiction scenarios.

The proposed approach is validated on five benchmark instances (T1–T5) obtained from the public dataset available at `https://github.com/sulmanqureshi/FacilityLocationDataset`. These instances vary in network size and interdiction budget k, providing a diverse testbed for evaluating solution performance. Figure 1 presents the percentage increase in total system cost after applying the worst-case interdiction identified by the GA, thus offering a quantitative measure of network vulnerability under strategic attack.

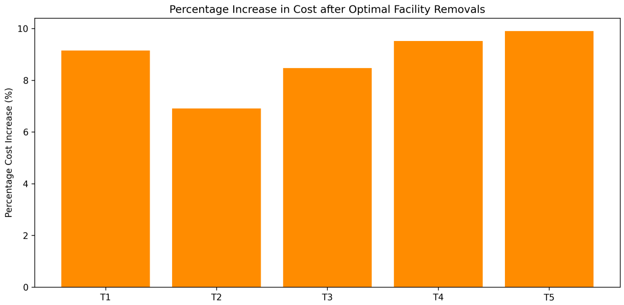

FIG. 1: Percentage Cost Increase for Instances T1-T5

4 Conclusion and Perspectives

We presented a model to identify the k most critical facilities by maximizing post-disruption costs, solved via a Genetic Algorithm. This approach effectively quantifies network vulnerability and pinpoints critical assets. Future work will explore larger case studies, compare metaheuristics, and incorporate stochastic failures or hardening budgets.

References

[1] Church, R. L., Scaparra, M. P., & Middleton, R. S. (2004). Identifying critical infrastructure: The median and covering facility interdiction problems. *Annals of the Association of American Geographers*, 94(3), 491-502.