Multi-item uncapacited lot-sizing problems with minimum order quantity, inventory bounds and perishability

Lucas Berterottière[1], Simon Bussy[2], Adeline Fermanian[2], Massil Hihat[2], Safia Kedad-Sidhoum[3]

[1] LPSM, Sorbonne Université, UMR CNRS 8001, Paris, France
lucas.berterottiere@sorbonne-universite.fr
[2] LOPF, Califrais' Machine Learning Lab, Paris, France
{simon.bussy, adeline.fermanian, massil.hihat}@califrais.fr
[3] CNAM Paris, CEDRIC, France
safia.kedad_sidhoum@cnam.fr

Keywords : *lot-sizing, minimum order quantity, perishability, inventory bounds, Lagrangian relaxation.*

1 Introduction

In this abstract, we adress a multi-item uncapacitated lot-sizing problem motivated by a real supply chain problem at Califrais, the digital operator of Rungis (the largest fresh product market in the world). Depending on demand, the company orders products from suppliers to meet customer needs. These suppliers set a minimum order quantity that must be respected. The commands are then stored in a warehouse with limited capacity. As the products involved are fresh, they cannot be distributed after a certain lifespan.

Several aspects of the considered problem have been studied independently in the literature. Polynomial-time algorithms have been proposed for the single-item lot-sizing problem with a minimum order quantity in [3, 5]. A single-item lot-sizing problem with an inventory bound is studied in [2]. The multi-item version of the problem has been proven NP-hard in [1]. Perishability constraints have been considered in two different ways: continuous deterioration [4] and complete deterioration after a certain lifetime [6].

2 Problem statement

The problem under investigation is an uncapacitated lot-sizing problem with an inventory bound, lead time, perishability, and a minimal order quantity. The objective of the problem is to determine a production plan for N items from the set $\mathcal{I} = \{1, \ldots, N\}$ over T periods from the set $\mathcal{T} = \{1, \ldots, T\}$. The demand for each item $i \in \mathcal{I}$ in period $t \in \mathcal{T}$ is denoted by d_{it}. This demand must be fully satisfied through orders. Two ordering modes are considered: the *standard* mode and the *urgent* mode. In the *standard* mode, there is a lead time l_i between the ordering period of an item i and the period when it is in inventory. Placing an order for item i in period t incurs a cost p_{it} per unit of item i ordered. Moreover, a minimum order quantity $q^k_{\min}$ must be respected for all products supplied by supplier k. The *urgent* mode allows items to be received on the ordering period. For an item i ordered in period t, this mode incurs a unit ordering cost p'_{it}, with $p'_{it} > p_{it}$.

An item i can be stored between its arrival period and its consumption period. It occupies a storage volume V_i in a warehouse with a capacity of $S_{\max}$. Each item has a maximum selling lifetime u_i after its arrival at the warehouse, beyond which it cannot be consumed. At the beginning of the planning horizon, an initial inventory I_{0i} is available for each item i.

Let x_{it} and x'_{it} be the quantities of item i ordered in the period t, and s_{it} be the quantity of item i stored in the end of period t. A mixed-integer linear programming model of the problem will be presented at the conference.

3 Solution method

As stated above, the multi-item uncapacitated lot-sizing problem with inventory bound is NP-hard in the strong sense. Thus, relaxing inventory bound constraints allows us to decompose the problem into several polynomial problems. A Lagrangian relaxation heuristic is therefore proposed to solve the problem as described in Algorithm 1.

Algorithm 1: Lagrangian Relaxation Heuristic

Input: order prices $\{p_{it}\}_{i\in\mathcal{I},t\in\mathcal{T}}$, holding prices $\{h_{it}\}_{i\in\mathcal{I},t\in\mathcal{T}}$, demand $\{d_{it}\}_{i\in\mathcal{I},t\in\mathcal{T}}$
Output: Lower bound $L(\lambda)$, upper bound UB, order quantity $\{\hat{x}_{it}\}_{i\in\mathcal{I},t\in\mathcal{T}}$, stock quantity $\{\hat{s}_{it}\}_{i\in\mathcal{I},t\in\mathcal{T}}$
Initialize multipliers $\lambda \geq 0$;
repeat
  Compute lower bound $L(\lambda)$ using polynomial DP algorithm ;
  Build a feasible solution $\hat{x}$ (repair heuristic) ;
  Compute upper bound value $UB = c^T\hat{x}$;
  Update multipliers λ ;
until $UB - L(\lambda) \leq \epsilon$;
return $L(\lambda)$, UB, $\hat{x}$

Two cases are considered, where relaxed problems are solved with different polynomial algorithms:

1. One item per supplier: a polynomial dynamic programming (DP) algorithm is used to determine the best policy for each item while ensuring minimum order quantities and perishability constraints. The algorithm provides an optimal solution in $O(T^3)$.

2. Several items per supplier: The demand for the different items from the supplier is aggregated, and the algorithm used in case 1 is applied. The resulting aggregated solution is then disaggregated using a minimum-cost flow algorithm. The algorithm is proven to compute an optimal solution in polynomial time.

References

[1] Ayse Akbalik, Bernard Penz, and Christophe Rapine. Multi-item uncapacitated lot sizing problem with inventory bounds. *Optimization Letters*, 9(1):143–154, 2015.

[2] Alper Atamtürk and Simge Küçükyavuz. An o(n2) algorithm for lot sizing with inventory bounds and fixed costs. *Operations Research Letters*, 36(3):297–299, 2008.

[3] Bertrand Hellion, Fabien Mangione, and Bernard Penz. A polynomial time algorithm for the single-item lot sizing problem with capacities, minimum order quantities and dynamic time windows. *Operations Research Letters*, 42(8):500–504, 2014.

[4] Vernon Ning Hsu. Dynamic economic lot size model with perishable inventory. *Management Science*, 46(8):1159–1169, 2000.

[5] Chung-Yee Lee. Inventory replenishment model: lot sizing versus just-in-time delivery. *Operations Research Letters*, 32(6):581–590, 2004.

[6] Mehmet Önal, H Edwin Romeijn, Amar Sapra, and Wilco Van Den Heuvel. The economic lot-sizing problem with perishable items and consumption order preference. *European Journal of Operational Research*, 244(3):881–891, 2015.